Northern Beaches Secondary College
Manly Campus

2023

Higher School Certificate Trial Examination

Mathematics Extension 1

General Instructions	• Reading time – 5 minutes • Working time – 2 hours • Write using black pen • NESA approved calculators may be used • A reference sheet is provided with this paper • For questions in Section II, show relevant mathematical reasoning and/or calculations
Total Marks: 70	**Section I – 10 marks (pages 2–6)** • Attempt Questions 1-10 • Allow about 15 minutes for this section • Answer on Multiple Choice Answer Sheet provided **Section II – 60 marks (pages 7–14)** • Attempt Questions 11–14 • Allow about 1 hour and 45 minutes minutes for this section • Answer each question in a separate booklet

Section I

10 marks

Attempt Questions 1 – 10

Allow about 15 mins for this section.

Use the multiple-choice answer sheet for Questions 1 – 10.

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1. What is the domain and range of $y = 4\cos^{-1}\left(\frac{3x}{2}\right)$.

A. Domain: $-\frac{2}{3} \leq x \leq \frac{2}{3}$

Range: $-4\pi \leq y \leq 4\pi$

B. Domain: $-\frac{3}{2} \leq x \leq \frac{3}{2}$

Range: $-4\pi \leq y \leq 4\pi$

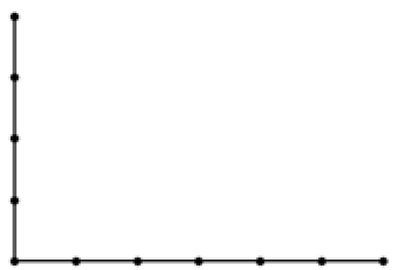
C. Domain: $-\frac{2}{3} \leq x \leq \frac{2}{3}$

Range: $0 \leq y \leq 4\pi$

D. Domain: $-\frac{2}{3} \leq x \leq \frac{2}{3}$

Range: $0 \leq y \leq 4$

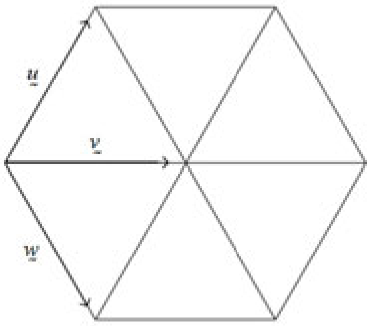
2. The diagram shows eleven points lying on two straight line intervals.



How many quadrilaterals can be formed using the points as vertices?

A. 90 B. 210 C. 290 D. 330

3. Six equilateral triangles form a hexagon with side lengths of 4 cm.
The vectors $\underset{\sim}{u}$, $\underset{\sim}{v}$ and $\underset{\sim}{w}$ are shown in the diagram.



NOT TO SCALE

Which of the following is the value of $\underset{\sim}{u} \cdot (\underset{\sim}{u} + \underset{\sim}{v} + \underset{\sim}{w})$?

A. 8

B. 16

C. 32

D. 48

4. Let random variable X be the number of successes in n Bernoulli trials with probability of success p, and let $q = 1 - p$.
Define $P(X = r)$ as the probability of r successes in n trials, where $0 \le r \le n$.
Which of the following is NOT always true?

A. $P(X = r) = {}^n\mathbf{C}_r \, p^r \, q^{n-r}$

B. $P(1 < X < 3) = {}^n\mathbf{C}_2 \, p^2 \, (1-p)^{n-2}$, given $n \ge 3$

C. $P(X \ge 1) = 1 - (1-p)^n$

D. $P(X = r) = {}^n\mathbf{C}_r \, p^r \, (1-p)^{n-r} = {}^n\mathbf{C}_{n-r} p^{n-r}(1-p)^r$

5. Which expression is the result of the integral $\int \frac{3}{9 + x^2} \, dx$?

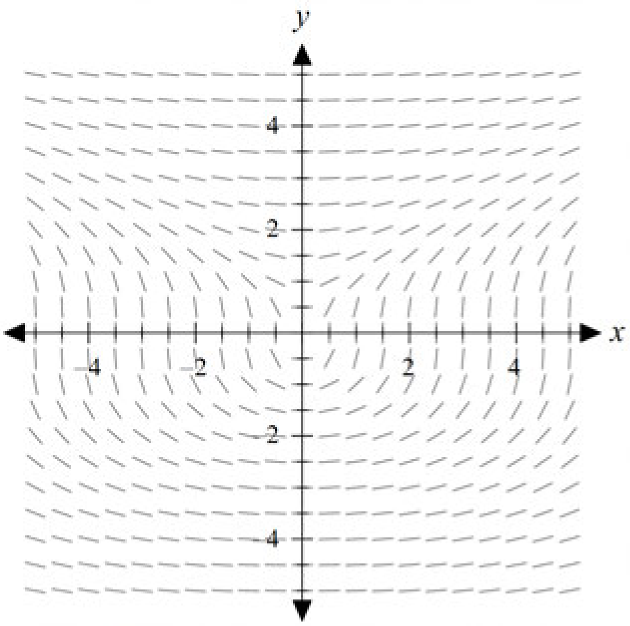
A. $\tan^{-1}\frac{x}{3} + c$

B. $\frac{1}{3}\tan^{-1}\frac{x}{3} + c$

C. $3\tan^{-1}\frac{x}{3} + c$

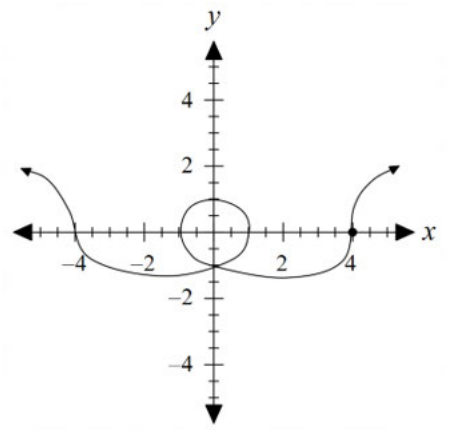
D. $\frac{1}{3}\tan^{-1}\frac{x}{9} + c$

6. The graph of the direction field of a differential is shown below.

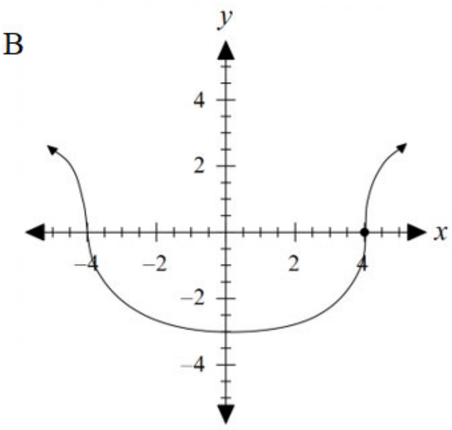


Which of the following best represents the particular solution which passes through the point (4, 0)?

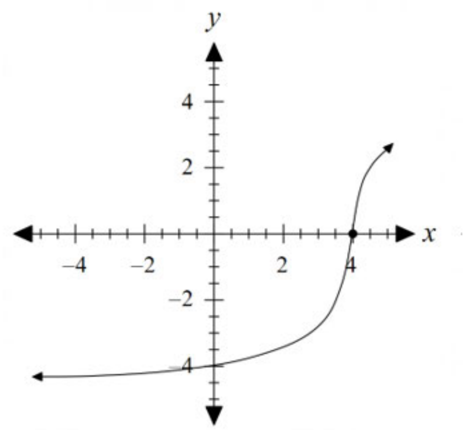
A



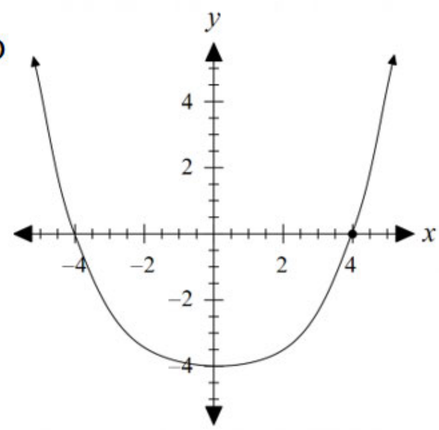
B



C



D



7. What is the term independent of x in the expansion of $\left(x^3 + \frac{2}{x}\right)^{20}$?

A. ${}^{20}\mathbf{C}_{10}\ 2^{10}$

B. ${}^{20}\mathbf{C}_{4}\ 2^{16}$

C. ${}^{20}\mathbf{C}_{5}\ 2^{25}$

D. ${}^{20}\mathbf{C}_{5}\ 2^{15}$

8. Two teams of 3 players each and an umpire are to be formed from seven people. If two of the people cannot be on the same team, which of the following is the number of ways the teams can be formed?

A. 100

B. 50

C. 140

D. 70

9. What does the integral $\int_0^{\frac{\pi}{7}} \sin 3x\ \sin 4x\ dx$ simplify to?

A. $2 \sin \frac{\pi}{7}$

B. $2 \sin \frac{\pi}{7} - \frac{1}{7}$

C. $\frac{1}{2} \sin \frac{\pi}{7}$

D. $\frac{1}{2} \sin \frac{\pi}{7} - \frac{1}{7}$

10. Which of the following functions is NOT a solution to the differential equation?

$y'' + 3y' - 4y = 0$?

A. $y = e^{-4x}$

B. $y = e^{x}$

C. $y = e^{5x}$

D. $y = 0$

Section II

60 marks

Attempt Questions 11 – 14

Allow about 1 hour and 45 minutes for this section.

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 Marks) Use a SEPARATE writing booklet.

a) Find the exact value $\cos^{-1}\left(\sin\frac{4\pi}{3}\right)$. **1**

b) A Mathematics department consists of 12 teachers. Nine of the teachers wear glasses and three of the teachers do not wear glasses. Five of these teachers go out for dinner together.
What is the probability there will be more teachers who wear glasses than teachers who do not wear glasses in the group who go out for dinner? **2**

c) If the polynomials $P(x) = 2x^3 + mx^2 + x - 3$ and $Q(x) = x^2 + 6x - 5$ have the same remainder when divided by $x + 1$, find the value of m. **2**

d) The area under the curve $y = \log_e(x-2)$ which is bounded by the x axis and the line $x = 7$ is rotated about the x-axis.

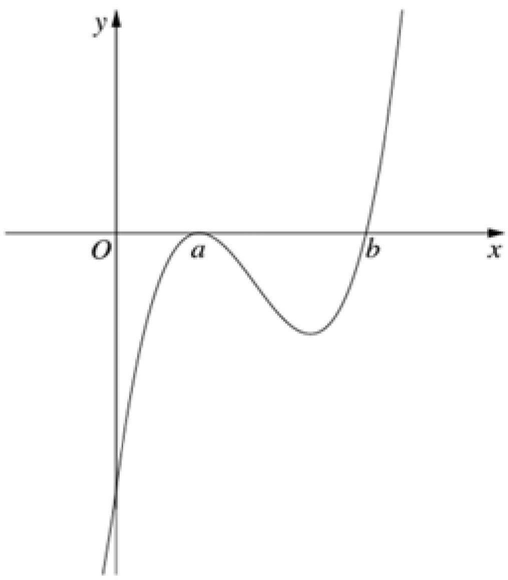
Use the trapezoidal rule with three function values to find the approximate volume of the solid of revolution formed. Express your answer to 2 significant figures. **3**

e) (i) Write down the expansion of $(1-x)^5$. **1**

(ii) Hence, or otherwise find the term of x^2 in $(2x-3)^2\,(1-x)^5$. **2**

Question 11 continues on the next page

f) The graph of the function $y = f(x)$ is shown



Draw a separate one-third of a page in your answer booklet the diagram for the graph of $y^2 = f(x)$. **2**

g) A random variable X defines a binomial distribution where $E(X) = 15$ and $Var(X) = 3.75$.
Find the values of n and p. **2**

End of Question 11

Question 12 (15 Marks) Use a SEPARATE writing booklet.

a) Differentiate $y = 4\tan^{-1}\frac{x}{2}$ **1**

b) Quality control for the manufacturing of bolts is carried out by taking a random selection of 15 bolts from a batch of 10 000. Empirical data has determined that 10% of bolts are defective. If three or more bolts in the sample are found to be defective, that batch is rejected.

(i) Find the probability that the batch is rejected, correct to 3 decimal places. **2**

(ii) The cost to produce the batch of 10 000 is \$20. Each batch is then either sold for \$40 or it is sold as scrap metal for \$5. **1**

Find the expected value of the batch of 10 000.

c) (i) Show that $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$. **2**

(ii) Hence, if a particle which is moving along the x-axis has velocity given by $v(t) = \sin^3 t$, find the position function of the particle with respect to time, given the particle starts at the origin. **2**

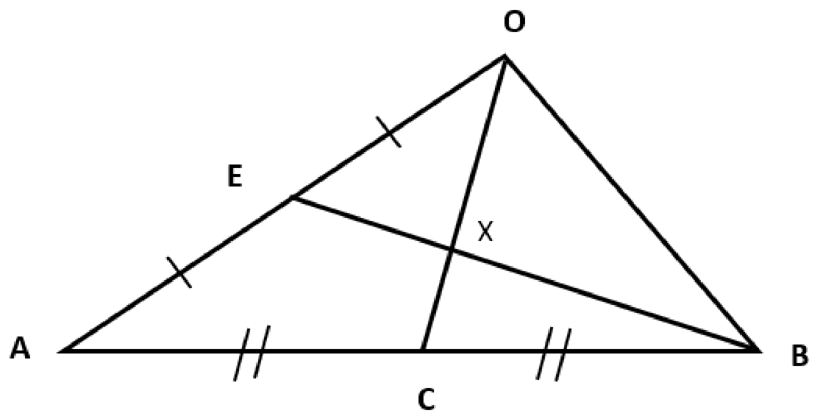
d) Prove $\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3 = \pi$. **2**

e) Calculate the exact area bounded by the curve $y = e^x$, the y-axis and the line $y = e^2$. **2**

Question 12 continues on the next page

f) The diagram shows triangle *AOB*, where $\overrightarrow{OA} = \underset{\sim}{a}$ and $\overrightarrow{OB} = \underset{\sim}{b}$.

OC and OE are medians which intersect at *X* as shown. E bisects $\overrightarrow{OA}$ while C bisects $\overrightarrow{AB}$.



(i) Find $\overrightarrow{OC}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$. **1**

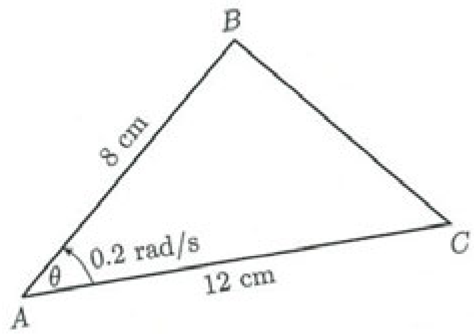
(ii) If $\overrightarrow{OC}$ is perpendicular to $\overrightarrow{AB}$, prove that triangle *AOB* is isosceles. **2**

End of Question 12

Question 13 (15 Marks) Use a SEPARATE writing booklet.

a) Solve the differential equation $\dfrac{dy}{dx} = x - x\sin^2 y$, given that the graph of the solution passes through the point (1, 0) **2**

b) In triangle ABC, $\angle BAC$ is increasing at a constant rate of 0.2 radians/s.
Find the rate of change in the area ΔABC when $\theta = \dfrac{\pi}{3}$. **2**



c) Use sums to products of trigonometric ratios to solve the equation.

$\cos x + \cos 2x + \cos 3x = 0$ where $0 \leq x \leq 2\pi$ **3**

You may use the information on page 15 to answer Question 13 d)

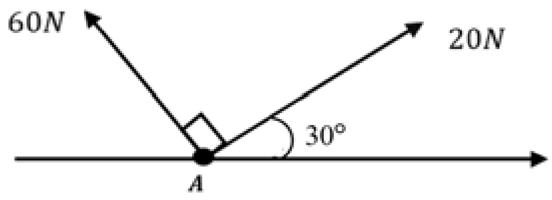
d) When all votes were counted at an election, it was found that 52% of voters chose Candidate A.
As votes were still being counted, a news agency conducted exit polling wherein they asked 1000 randomly selected voters as they were leaving the voting centre which candidate they voted for.
Assume all voters were equally likely to be selected to participate and all answered truthfully.

(i) What was the probability that the news agency's sample proportion who voted for Candidate A would be less than 0.5? **3**

(ii) Suppose the proportion of the voters who chose Candidate A is now 40%. What is the smallest sample size such that the standard deviation is less than 0.01? **2**

Question 13 continues on the next page

e) Forces of 60N and 20N act on an object, considered point A, as shown in the diagram.



(i) Find the sum of the resolved parts of the forces in the horizontal direction. **1**

(ii) The vector sum of these forces acting on the object at point A is called the resultant force.
Find the resultant force vector in the form $x\underset{\sim}{i} + y\underset{\sim}{j}$. **2**

End of Question 13

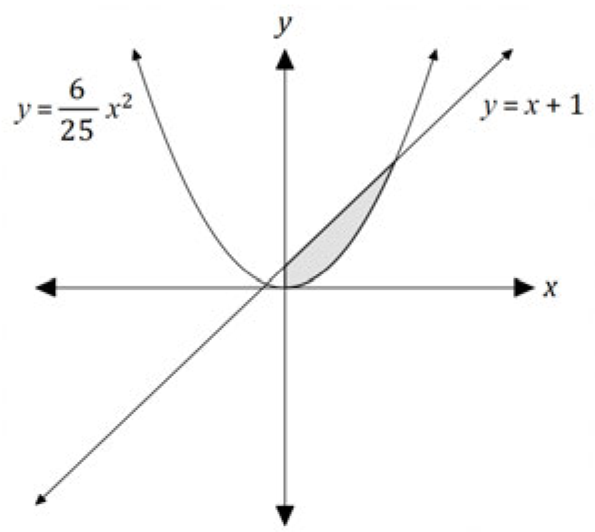
Question 14 (15 Marks) Use a SEPARATE writing booklet.

a) A frozen dinner has a temperature of 0° C and is placed in an oven with an ambient temperature of 180° C. The rate at which the dinner warms per minute is proportional to the difference between the dinner's temperature, T, and the ambient temperature. That is, $\frac{dT}{dt} = k(T - 180)$ where k is a constant.

It is given that the temperature of the dinner at time t is given by a function of the form $T = B + Ae^{kt}$ where A and B are constants.

If the dinner's temperature is 25° C after 15 minutes, find the constants A, B, and k, showing mathematical reasoning for each, and find the rate of change of the dinner's temperature when it is 100° C. **4**

b) The graphs of $y = x + 1$ and $y = \frac{6}{25}x^2$ are shown on the axes below and the area enclosed by the y-axis and the two functions is shaded.



Find the exact volume of the solid of revolution formed by rotating the shaded area about the x-axis. **3**

c) Given $7 + 12x - 4x^2 = 16 - (2x - 3)^2$.

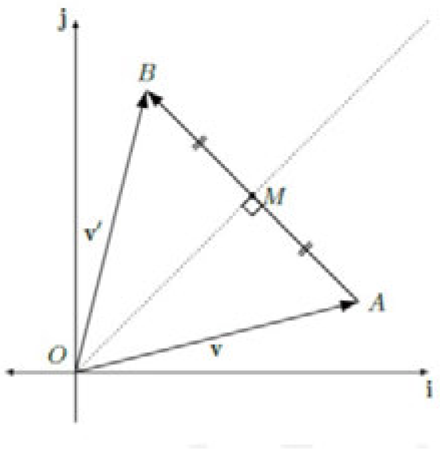
Find the value of $\int_0^2 \frac{1}{\sqrt{7 + 12x - 4x^2}}\,dx$. Leave your answer in exact form. **2**

Question 14 continues on the next page

Question 14 (continued)

d) The diagram shows the vector $v = \begin{pmatrix} a \\ b \end{pmatrix}$.

v' is its reflection about the dotted line, $y = x$. M bisects AB and lies on $y = x$.



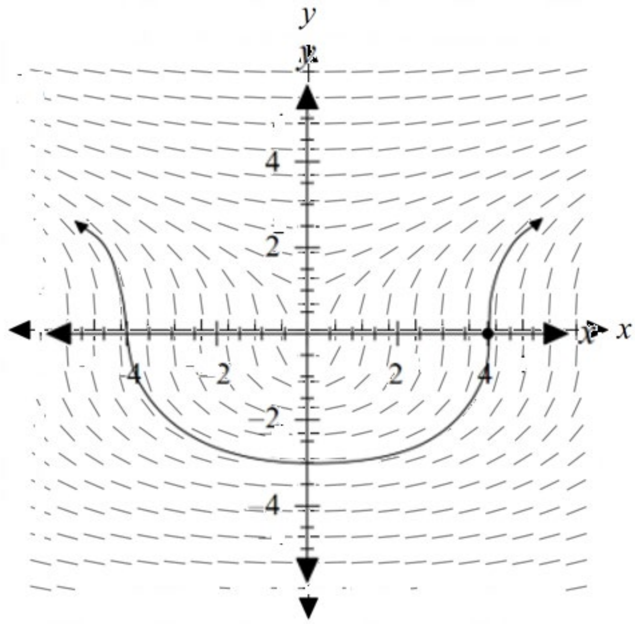
Using vector projections, show that $v' = \begin{pmatrix} b \\ a \end{pmatrix}$. **3**

e) Use mathematical induction to prove that

$$\frac{1}{n(n+1)} + \frac{1}{(n+1)(n+2)} + \frac{1}{(n+2)(n+3)} + \ldots + \frac{1}{(2n-1)\,2n} = \frac{1}{2n}$$

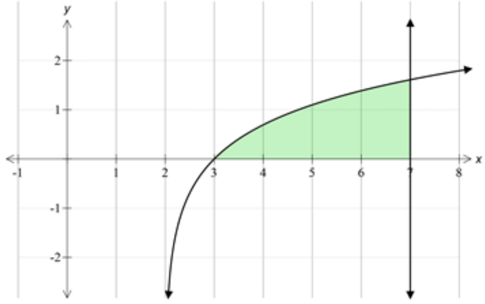
for all integers $n \geq 1$. **3**

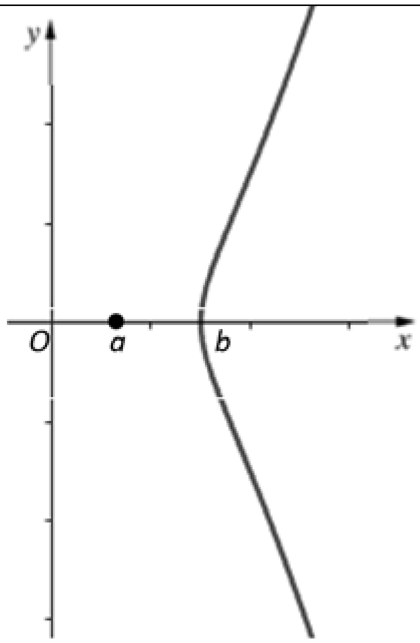
End of paper

Question	Solution	Marking Guideline
1	$-1 \le \frac{3x}{2} \le 1$ $-\frac{2}{3} \le x \le \frac{2}{3}$ domain $0 \le \cos^{-1}\left(\frac{3x}{2}\right) \le \pi$ $0 \le 4\cos^{-1}\left(\frac{3x}{2}\right) \le 4\pi$ range	C
2	${}^4C_2 \times {}^6C_2 = 90$ corner point is not relevant	A
3	$\underset{\sim}{u} . (\underset{\sim}{u} + \underset{\sim}{v} + \underset{\sim}{w})$ $= \lvert\underset{\sim}{u}\rvert \, \lvert\underset{\sim}{u} + \underset{\sim}{v} + \underset{\sim}{w}\rvert \cos 60°$ $= 4 \times 8 \times \frac{1}{2}$ $= 16$	B
4	Option D is only true when **p = q = 0.5**	D
5	$3 \times \frac{1}{3}\tan^{-1}\frac{x}{3} + C$ $= \tan^{-1}\frac{x}{3} + C$	A
6		B

7	General term $^{20}C_r\ (x^3)^{20-r}\left(\frac{2}{x}\right)^r$ $= {}^{20}C_r\ x^{60-3r}\ \frac{2^r}{x^r}$ $= {}^{20}C_r\ 2^r\ x^{60-4r}$ $\therefore 60 - 4r = 0$ $r = 15$ $= {}^{20}C_{15}\ \times\ 2^{15}$ $= {}^{20}C_5\ \times\ 2^{15}$	D
8	Case 1: one of the two people is the umpire, then form the teams $^2C_1\ \times\ \frac{^6C_3\ \times\ ^3C_3}{2} = 20$ Have to divide by 2 as teams are indistinguishable . Case 2: separate the two players into the two teams then sort the other players as a ref and teams. $^5C_1\ \times\ ^4C_2\ \times\ ^2C_2 = 30$ Total number = 20 + 30 = 50	B
9	$= \frac{1}{2}\int_0^{\frac{\pi}{7}} (\cos x - \cos 7x)\ dx$ $= \frac{1}{2}\left[\sin x\ -\ \frac{1}{7}\sin 7x\right]_{\frac{\pi}{7}}^{0}$ $= \frac{1}{2}\sin\frac{\pi}{7} - \frac{1}{14}\sin\pi - 0$ $= \frac{1}{2}\sin\frac{\pi}{7}$	C
10	Testing each option, clearly C is the solution. $y = e^{5x}$ $y' = 5e^{5x}$ $y'' = 25e^{5x}$ $25e^{5x} + 15e^{5x} - 4e^{5x} \neq 0$	C
11a	Let $\theta = \cos^{-1}\left(\sin\frac{4\pi}{3}\right)$ $\cos\theta = \sin\frac{4\pi}{3}$ $= -\frac{\sqrt{3}}{2}$ $\theta = \pi - \frac{\pi}{6}$ $(\cos\theta < 0)$ $= \frac{5\pi}{6}$	1 mark correct solution

	On this page, just above it states "your solutions should include relevant mathematical reasoning and/or calculations". **Students MUST show working.**	
11b	Total number of ways to choose 5 teachers = ${}^{12}_{5}C = 792$ Number with more teachers with glasses = Number 5 glasses + Number 4 glasses + Number 3 glasses $= \binom{9}{5} + \binom{9}{4} \times \binom{3}{1} + \binom{9}{3} \times \binom{3}{2}$ $= 756$ Probability $= \frac{756}{792} = \frac{21}{22}$	2 marks correct solution. 1 mark total number or correct working for probability with one error.
11c	By Remainder Theorem, $Q(-1) = (-1)^2 + 6(-1) - 5$ $= -10$ $P(-1) = 2(-1)^3 + m(-1)^2 + (-1) - 3$ $= -6 + m$ $\therefore -6 + m = -10$ $\therefore m = -4$ Students MUST show working.	2 marks correct solution 1 mark progress with ONLY one error (finds Q(-1) and attempts to equate to P(-1))
11d		3 marks correct solution 2 marks correct solution but not 2 s.f OR correct progress with ONLY one error 1 mark Correctly applies the trapezoidal rule or equivalent progress

	 $$Vol = \pi \int_3^7 \log_e (x-2)^2 \, dx$$ 3 function values give 2 sub-intervals <table><tr><td>x</td><td>3</td><td>5</td><td>7</td></tr><tr><td>y^2</td><td>0</td><td>$(\log_e 3)^2$</td><td>$(\log_e 5)^2$</td></tr></table> Trapezoidal Rule: $$Vol = \pi\left(\frac{7-3}{2x2}\{0 + (\log_e 5)^2 + 2((\log_e 3)^2)\}\right)$$ $$Vol = \pi((\log_e 5)^2 + 2((\log_e 3)^2))$$ $$Vol = 15.72 \, u^3$$ $$Vol = 16 \, u^3 \text{ to 2 sig.fig}$$	
11e i	$(1-x)^5 = (1+(-x))^5$ $= \binom{5}{0}(-x)^0 + \binom{5}{1}(-x)^1 + \binom{5}{2}(-x)^2 + \binom{5}{3}(-x)^3 + \binom{5}{4}(-x)^4 + \binom{5}{5}(-x)^5$ $= 1 - 5x + 10x^2 - 10x^3 + 5x^4 - x^5$	1 mark correct solution
11e ii	$(2x-3)^2 = 4x^2 - 12x + 9$ The term in $x^2 = 4x^2 \times 1 + (-12x) \times (-5x) + 9 \times 10x^2$ $= 154x^2$ **Students MUST show working.** 154 is the coefficient not the term.	2 marks correct solution 1 mark correct progress with ONLY one error

11f		2 marks correct graph 1 mark, only one side of square root plotted or equivalent merit.
11g	$E(X) = np$ $15 = np$ $Var(X) = np(1-p)$ $3.75 = 15(1-p)$ $p = 1 - 0.25 = 0.75$ $n = \frac{15}{0.75} = 20$	2 marks correct solution 1 mark correct progress with ONLY one error

12a	$y' = \frac{4}{1 + \left(\frac{x^2}{4}\right)} \times \frac{1}{2}$ $= \frac{8}{4 + x^2}$	1 mark correct solution **MUST BE FULLY SIMPLIFIED**
12bi	$P(X \geq 3) = 1 - (P(X = 0) + P(X = 1) + P(X + 2))$ $= 1 - \left(0.9^{15} + {}^{15}\mathbf{C}_1 \times 0.1 \times 0.9^{14} + {}^{15}\mathbf{C}_2 \times 0.1^2 \times 0.9^{13}\right)$ $= 0.184$	2 marks correct solution 1 mark **correct approach** with ONLY one error in calculation
12bii	Expected Value = $0.816 \times 40 + 0.184 \times 5 - 20$ $= \$13.56$	1 mark correct solution **showing calculation**
12ci	$\sin(2\theta + \theta) = \sin 2\theta\cos\theta + \cos 2\theta \sin\theta$ $= 2\sin\theta(1 - \sin^2\theta) + \sin\theta(1 - 2\sin^2\theta)$ $= 2\sin\theta - 2\sin^3\theta + \sin\theta - 2\sin^3\theta$ $= 3\sin\theta - 4\sin^3\theta$	2 marks correct solution 1 mark correct progress **working only one side at a time** with ONLY one error
12cii	$x = \int \sin^3 t \, dt$ $= \frac{1}{4}\int (3\sin \quad t\text{-}\sin 3t)dt$ $= \frac{1}{4}\left[-3\cos t + \frac{1}{3}\cos 3t\right] + c$ $\therefore \; 0 = \frac{1}{4}\left[-3\cos 0 + \frac{1}{3}\cos 0\right] + c$ $c = \frac{2}{3}$ $\therefore \; x = -\frac{3}{4}\cos t + \frac{1}{12}\cos 3t + \frac{2}{3}$	2 marks correct solution 1 mark correct progress with ONLY one error **MUST show calculation of constant**

<table>
<tr><td>12d</td><td>

$$LHS = \frac{\pi}{4} + \tan^{-1}2 + \tan^{-1}3$$

let $\alpha = \tan^{-1}2,\ \beta = \tan^{-1}3$

$$LHS = \frac{\pi}{4} + \alpha + \beta$$

$$\text{now } \tan\left(\frac{\pi}{4} + (\alpha + \beta)\right)$$

$$= \frac{\tan\frac{\pi}{4} + \tan(\alpha + \beta)}{1 - \tan\frac{\pi}{4}\tan(\alpha + \beta)}$$

$$= \frac{1 + \tan(\alpha + \beta)}{1 - \tan(\alpha + \beta)}$$

$$= \frac{1 + \frac{\tan\alpha + \tan B}{1 - \tan\alpha\tan B}}{1 - \frac{\tan\alpha + \tan B}{1 - \tan\alpha\tan B}}$$

$$= \frac{1 + \frac{2+3}{1-2\times 3}}{1 - \frac{2+3}{1-2\times 3}}$$

$$= \frac{1-1}{1+1}$$

$$= 0$$

$$RHS = \pi$$

and $\tan\pi = 0$

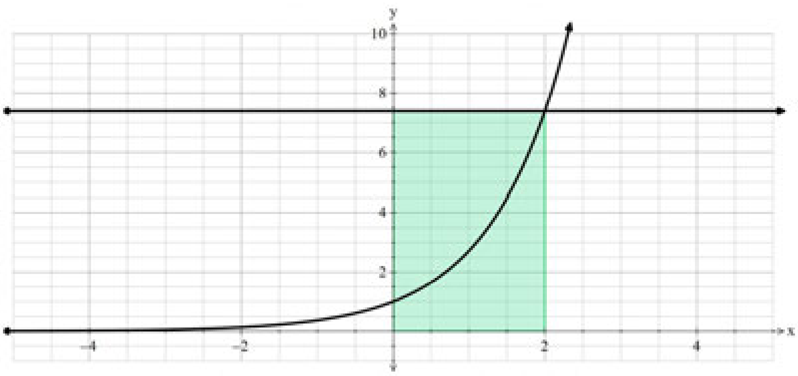
$\therefore$ LHS=RHS

</td><td>2 marks correct solution

1 mark correct approach working only one side at time

Did not move terms across equal sign

MUST demonstrate use of tan compound angle to achieve a numerical value</td></tr>
<tr><td>12e</td><td>



Area=rectangle - $\int_0^2 e^x\,dx$

</td><td>2 marks correct solution

1 mark correct attempt with ONLY one error
Did not simplify Q</td></tr>
</table>

	$= (2 \times e^2) - \int_0^2 e^x\, dx$ $= 2e^2 - [e^x]^0{,}2$ $= 2e^2 - (e^2 - e^0)$ $= 2e^2 - e^2 + 1$ $= e^2 + 1\ u^2$	
12fi	$OA + AB = OB$ $AB = b - a$ $CB = \frac{1}{2}(b - a)$ now $OC + CB = OB$ $OC = b - \frac{1}{2}(b - a)$ $OC = b - \frac{1}{2}b + \frac{1}{2}\text{a}$ $= \frac{1}{2}a + \frac{1}{2}b$ $= \frac{1}{2}(a + b)$	1 mark correct solution showing all working and **FULLLY SIMPLIFIED**
12fii	$\overrightarrow{OC} \cdot \overrightarrow{AB} = 0$ $\left(\frac{1}{2\underset{\sim}{\mathbf{a}}} + \frac{1}{2\underset{\sim}{\mathbf{b}}}\right) \cdot (\underset{\sim}{\mathbf{b}} - \underset{\sim}{\mathbf{a}}) = 0$ $\frac{1}{2}\underset{\sim}{\mathbf{a}} \cdot \underset{\sim}{\mathbf{b}} - \frac{1}{2}\underset{\sim}{\mathbf{a}} \cdot \underset{\sim}{\mathbf{a}} + \frac{1}{2}\underset{\sim}{\mathbf{b}} \cdot \underset{\sim}{\mathbf{b}} - \frac{1}{2}\underset{\sim}{\mathbf{a}} \cdot \underset{\sim}{\mathbf{b}} = 0$ $\frac{1}{2}\lvert\underset{\sim}{\mathbf{b}}\rvert^2 - \frac{1}{2}\lvert\underset{\sim}{\mathbf{a}}\rvert^2 = 0$ $\lvert\underset{\sim}{\mathbf{b}}\rvert^2 = \lvert\underset{\sim}{\mathbf{a}}\rvert^2$ $\lvert\underset{\sim}{\mathbf{b}}\rvert = \lvert\underset{\sim}{\mathbf{a}}\rvert$ $\therefore$ sides lengths $\overrightarrow{OB}$ and $\overrightarrow{OA}$ are equal $\therefore \Delta AOB$ is isosceles	2 marks correct solution 1 mark correct progress using given info to show equal sides with ONLY one error **MUST SHOW ALL WORKING**

13a	$\frac{dy}{dx} = x(1 - \sin^2 y)$ $= x\cos^2 y$ $\frac{1}{\cos^2 y} dy = x\, dx$ $\int \sec^2 y\, dy = \int x\, dx$ $\tan y = \frac{1}{2}x^2 + c$ $\tan 0 = \frac{1}{2} + c$ $c = -\frac{1}{2}$ $\therefore \quad \tan y = \frac{1}{2}x^2 - \frac{1}{2}$ $y = \tan^{-1}\left(\frac{1}{2}x^2 - \frac{1}{2}\right)$	2 marks for correct working and solution. 1 mark for correct integral or equivalent merit.
13b	$A = \frac{1}{2}\text{ab}\sin\theta$ $A = 48\sin\theta$ $\frac{dA}{d\theta} = 48\cos\theta$ $\frac{dA}{dt} = \frac{dA}{d\theta} \times \frac{d\theta}{dt}$ $= 48\cos\theta \times 0.2$ when $\theta = \frac{\pi}{3}$ $\frac{dA}{dt} = 4.8$ Area changing at a rate of 4.8 $\text{cm}^2 s^{-1}$	2 marks for correct working and solution. 1 mark for correct derivative or equivalent merit.
13c	group $\cos x$ and $\cos 3x$ $\cos x + \cos 3x = 2\cos 2x \cos x$ (reference sheet) $\therefore 2\cos 2x \cos x + \cos 2x = 0$ $\cos 2x(2\cos x + 1) = 0$ $\cos 2x = 0 \quad \text{or} \quad \cos x = -\frac{1}{2}$ $2x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2} \quad \text{or} \quad x = \frac{2\pi}{3}, \frac{4\pi}{3}$ $\therefore \quad x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \frac{2\pi}{3}, \frac{4\pi}{3}$	3 marks for correct working and solution. 2 marks, finds some correct solutions 1 mark correctly expands the expression or

		equivalent progress.
13d i	$$\delta = \frac{\sqrt{1\,000 \times 0.52 \times 0.48}}{1\,000}$$ $= 0.0158$ $$z = \frac{0.5 - 0.52}{0.0158}$$ $= -1.27$ $P(z < -1.27) = 1 - 0.8980$ $= 0.102$ $= 10.2\%$	3 marks for correct working and solution 2 marks for correct standard deviation and z score 1 mark for correct standard deviation
13d ii	$$\frac{\sqrt{npq}}{n} < 0.01$$ $p = 0.4$ and $q = 0.6$ $\sqrt{0.24n} < 0.01n$ $0.24n < 0.0001n^2$ $0.24n - 0.0001n^2 < 0$ $0.24 - 0.0001n < 0$ since $n \neq 0$ $0.24 < 0.0001n$ $2\,400 < n$ $n = 2\,401$	2 marks for correct working and solution 1 mark for correct inequality or equivalent
13 e i	$20 \cos 30° + 60 \sin 120° = 10\sqrt{3} - 30$ newtons	1 mark
13 e ii	vertical direction $20\sin 30° + 60\sin 120°$ $= 10 + 30\sqrt{3}$ $\therefore \left(10\sqrt{3} - 30\right)\underset{\sim}{i} + \left(10 + 30\sqrt{3}\right)\underset{\sim}{j}$	2 marks for correct solution 1 mark for vertical direction.

14 a)	$T = B + Ae^{kt}$ $Ae^{kt} = T - B$ $\frac{dT}{dt} = kAe^{kt}$ $= k(T - B)$ $= k(T - 180)$ $\therefore B = 180$ $T = Ae^{kt} + 180$ $t = 0, T = 0$ $0 = Ae^0 + 180$ $\therefore A = -180$ $T = -180e^{kt} + 180$ $t = 15, T = 25$ $25 = -180e^{15k} + 180$ $\frac{31}{36} = e^{15k}$ $\frac{\ln 31}{36} = 15k$ $\therefore k = \frac{\frac{\ln 31}{36}}{15}$ $T = -180e^{kt} + 180 \; where \; k \approx -0.009969$ $T = 100, \frac{dT}{dt} = -0.009969(100 - 180)$ $= 0.79750258° \frac{C}{min}$	4 marks for all correct substitutions and final value with the correct calculations. 1 mark for each of the values A & B. 1 mark for value of k. 1 mark for the correct final rate.
Generally done well. Students should note that substitutions need to be shown and the exact value of k should be used in the final step. Transcription Errors (TE) were common in this question.		

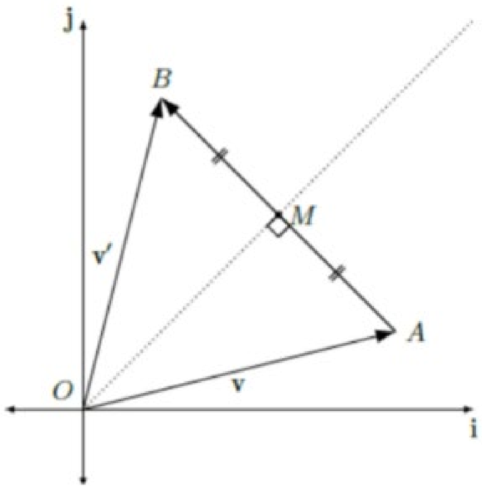
14 b)	$\frac{6}{25}x^2 = x+1$ $6x^2 - 25x - 25 = 0$ $(6x+5)(x-5) = 0$ $\therefore x = -\frac{5}{6}, x = 5$ $from\ graph, shaded\ between\ x = 0, x = 5$ $V = \pi\int_0^5 \left[(x+1)^2 - \left(\frac{6}{25}x^2\right)^2\right]dx$ $= \pi\left[\frac{(x+1)^3}{3} - \frac{\frac{36}{625}x^5}{5}\right]_0^5$ $= \pi\left[(72-36) - \left(\frac{1}{3} - 0\right)\right]$ $= \frac{107\pi}{3}\ u^3$	3 marks for correct boundaries, area between 0-5 and correct exact value integration 2 marks for correct integration with incorrect boundaries. 1 mark for boundaries. 1 mark for area with correct boundaries.
Most solutions had the points of intersection of graphs found correctly. Errors stemmed from using the points of intersection as boundaries of the integral, ie $-\frac{5}{6}\ to\ 5$. Few solutions had calculated $V = \pi\int\ \_0\hat{}5▒\ 〖((x + 1 - 6/25\ x\hat{}2\ )\hat{}2\ dx〗$ which was incorrect, and marks were not awarded. Students should remember to leave it in exact form, with cubic units.		

14 c)	$$\int_0^2 \frac{1}{\sqrt{7+12x-4x^2}}$$ $$=\int_0^2 \frac{2}{\sqrt{16-(2x-3)^2}}$$ $$=\frac{1}{2}[sin^{-1}(\frac{2x-3}{4})]_0^2$$ $$=\frac{1}{2}\left(sin^{-1}\left(\frac{1}{4}\right)-sin^{-1}\left(-\frac{3}{4}\right)\right)$$ $$=\frac{1}{2}\left(sin^{-1}\left(\frac{1}{4}\right)+sin^{-1}\left(\frac{3}{4}\right)\right)$$	2 marks for correct substitution of the given expression, and the final expression. 1 mark for correct inverse sin expression.

Most solutions had correct substitution as given in the question, however, some solutions lost the square root sign and simplified the question.
Some square root expressions incorrectly integrated to inverse tan, that did not gain any marks.
Solutions with this simplification also did not gain any marks.

$$\sqrt{16-(2x-3)^2}=4-(2x-3)$$

A few solutions correctly simplified the inverse sin of a negative value.
Students should remember to substitute the boundaries as some solutions had the second inverse sin expression as zero.

14 d)	 Let $\tilde{c}$ be the unit vector in the direction $\overrightarrow{OM}$ i.e., $\tilde{c} = \tilde{\imath} + \tilde{\jmath}$ $\overrightarrow{MA} = \perp \text{proj}_{\tilde{c}}\overrightarrow{OA}$ $= \overrightarrow{OA} - \text{proj}_{\tilde{c}}\overrightarrow{OA}$ $= \begin{pmatrix} a \\ b \end{pmatrix} - \frac{\begin{pmatrix} a \\ b \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix}}{1^2+1^2} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $= \begin{pmatrix} a \\ b \end{pmatrix} - \frac{a+b}{2} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$ $= \overrightarrow{OA} - 2\overrightarrow{MA}$ $= \begin{pmatrix} a \\ b \end{pmatrix} - 2\begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} a+b \\ a+b \end{pmatrix}$ $= \begin{pmatrix} a-2a+a+b \\ b-2b+a+b \end{pmatrix}$ $= \begin{pmatrix} b \\ a \end{pmatrix}$ $\therefore v' = \begin{pmatrix} b \\ a \end{pmatrix}$	3 marks for using projection correctly and completing the proof. 2 marks for identifying v' as the vector addition of $\overrightarrow{OA}$ and $\overrightarrow{OB}$ and successfully calculating $\overrightarrow{AB}$ using vector projection. 1 mark with success with one of the above.

Solutions required to use vector projection, hence, those that did not do correct use of projection did not receive any marks.
Students should use proper vector notation.
Solutions trying vector projection onto $\vec{AB}$ were not successful.
Better solutions defined the unit vector in $\overrightarrow{OM}$ direction.
Students who defined a vector projection but used the $y = x$ logic to swap a and b were not awarded full marks.

14 e)	Require to prove: $$\frac{1}{n(n+1)}+\frac{1}{(n+1)(n+2)}+\frac{1}{(n+2)(n+3)}+\cdots+\frac{1}{(2n-1)2n}=\frac{1}{2n}$$ for all integers $n \geq 1$. Consider $n = 1$. $L.H.S=\frac{1}{1(2)}=\frac{1}{2}$ $R.H.S=\frac{1}{2\times 1}=\frac{1}{2}$ $L.H.S = R.H.S$ $\therefore$ true for $n = 1$ Assume statement true for $n = k$, for $k \in \mathbb{Z}^+$ i.e. $$\frac{1}{k(k+1)}+\frac{1}{(k+1)(k+2)}+\frac{1}{(k+2)(k+3)}+\cdots+\frac{1}{(2k-1)2k}=\frac{1}{2k}$$ Prove true for $n = k + 1$ i.e. $$\frac{1}{(k+1)(k+2)}+\frac{1}{(k+2)(k+3)}+\cdots+\frac{1}{(2k-1)(2k)}+\frac{1}{2k(2k+1)}+\frac{1}{(2k+1)(2k+2)}=\frac{1}{2(k+1)}$$ Now using assumption, $L.H.S=\frac{1}{2k}-\frac{1}{k(k+1)}+\frac{1}{2k(2k+1)}+\frac{1}{(2k+1)(2k+2)}$ $=\frac{(k+1)(2k+1)-2(2k+1)+(k+1)+k}{2k(k+1)(2k+1)}$ $=\frac{2k^2+k}{2k(k+1)(2k+1)}=\frac{k(2k+1)}{2k(k+1)(2k+1)}=\frac{1}{2(k+1)}$ $\therefore$ true for $n = k + 1$ Hence, by mathematical induction, the statement is true for all positive integers $n \geq 1$	1 mark for proof for correct statements for $n = k$ & $n = k + 1$ 1 mark for incorporating $n = k + 1$ into proof, with all fractions correctly added & subtracted 1 mark for completed correct proof. More marks were awarded for the statements rather than n=1 proof, hence the mark was redirected.

Most solutions did not notice the subtraction of the term missing from $S(k)$ nor the extra term that was added from $S(k)$ to $S(k+1)$.

Some of those solution that included all four terms did not succeed with the addition/subtraction of 4 fractions. Those solution that simplified two fractions at a time had more success.

Solutions that did not show the correct simplification did not gain full marks.

Solutions that used sigma notation for the $S(k)$ and $S(k+1)$ were not able to see the pattern fully and only gained one mark.

Students should note that not all induction questions have one term added.